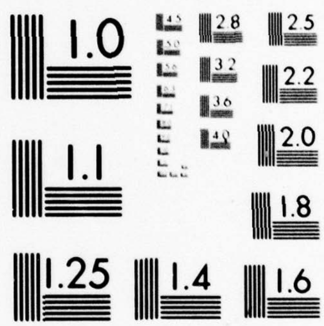
AD-A072 984 ARMY COMMUNICATIONS RESEARCH AND DEVELOPMENT COMMAND --ETC F/G 20/3
SCATTERING FROM PERIODIC SURFACES WITH SINUSOIDAL HEIGHT PROFIL--ETC(U)
JUN 79 G M WHITMAN, F SCHWERING

UNCLASSIFIED CORADCOM-79-4 NL

1 OF 1

AD
A072984

END
DATE
FILMED
9-79
DDC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

LEVEL II

12 B.S.

RESEARCH AND DEVELOPMENT TECHNICAL REPORT
CORADCOM-79-4

SCATTERING FROM PERIODIC SURFACES WITH SINUSOIDAL HEIGHT PROFILE - A THEORETICAL APPROACH-PART I: THEORY

G. M. Whitman
F. Schwering
COMMUNICATIONS SYSTEMS CENTER

June 1979

DISTRIBUTION STATEMENT
Approved for public release;
distribution unlimited.

CORADCOM
US ARMY COMMUNICATION RESEARCH & DEVELOPMENT COMMAND
FORT MONMOUTH, NEW JERSEY 07703

NOTICES

Disclaimers

The citation of trade names and names of manufacturers in this report is not to be construed as official Government indorsement or approval of commercial products or services referenced herein.

Disposition

Destroy this report when it is no longer needed. Do not return it to the originator.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 14 CORADCOM-79-4	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER 9
4. TITLE (and Subtitle) 6 Scattering from Periodic Surfaces with Sinusoidal Height Profile - A Theoretical Approach - Part I. Theory.		5. TYPE OF REPORT & PERIOD COVERED Final Report. 1 Jul[redacted]973 — 30 Jun[redacted]976,
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) 10 G. M. Whitman, F. Schwering		8. CONTRACT OR GRANT NUMBER(s) DAHC04-72-A-0001 Task Order 74-407
9. PERFORMING ORGANIZATION NAME AND ADDRESS CENCOMS, DRDCO-COM-RM-4 US Army Communications R&D Command Fort Monmouth, N.J. 07703		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 1L1 611102.H48,H2.11.01
11. CONTROLLING OFFICE NAME AND ADDRESS CENCOMS/CORADCOM Fort Monmouth, N.J. 07703		12. REPORT DATE 11 Jun[redacted]979
		13. NUMBER OF PAGES 23 (12) 28 p.
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Release; Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Periodic surface scattering; Sinusoidal height profile; Plane wave scattering; Space harmonics; Extended boundary condition; Rayleigh type Wood anomalies

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A theory of scattering by periodic metal surfaces is presented which utilizes the physical optics approximation to determine the current distribution in the metal surface in first order, but modifies this approximate distribution by multiplication with a Fourier series whose fundamental period is that of the surface profile (Floquet's theorem). The coefficients of the Fourier series are determined from the extended boundary condition that the field radiated by the current distribution into the lower (shielded) half-space must cancel the primary plane wave in this space range. The theory reduces (Cont'd)

next page

410 489

JOB

20. ABSTRACT (Cont'd)

the scatter problem to the familar task of solving a linear system. For certain basic topics of surface profiles, including the sinusoidal profile considered here, the coefficients of the linear system are obtained as closed form expressions in well-known functions (Bessel functions for sinusodial profiles and exponential functions for piece-wise linear profiles). The theory is thus amenable to efficient computer evaluation.

In part I of this report the theory is presented in detail; part II reports on numerical results obtained by computer evaluation of the theory.

TABLE OF CONTENTS

List of Figures

SCATTERING BY PERIODIC SURFACES WITH SINUSOIDAL HEIGHT PROFILE-A THEORETICAL APPROACH

PART I: THEORY

1. INTRODUCTION

Many of the procedures used to study scattering from periodic surfaces are based on the original approach used by Lord Rayleigh in 1907 [1] in which a discrete spectrum of outgoing plane waves is assumed for the scattered field. Difficulties occur in applying boundary conditions since this form of the scattered field does not necessarily apply at the scatter surfaces [2,3]. Other approaches to study scattering from periodic surfaces include perturbation techniques [4,5] variational methods [6] and the physical optics or Kirchhoff approximation [7,8,9]. All of these methods are of limited validity. Only recently, with the availability of high speed computers, has the evaluation of rigorous formulations of these problems become practical and

---

[1] Lord Rayleigh, The Theory of Sound, Vol. II, Dover Publications, New York (1945).

[2] Lippmann, B. A. (1953), "Note on the Theory of Gratings", J. Opt. Soc. Amer. 43, 408.

[3] Uretsky, J. L. (1965). "The Scattering of Plane Waves from Periodic Surfaces," Ann. Phys., 33, 400-427.

[4] Miles, J. W. (1954), "On Nonspecular Reflection at a Rough Surface", J. Acout. Soc. Amer., 26, 191-199.

[5] Katsenelenbaum, B. A. (1955), "The Perturbation of an Electromagnetic Field for Small Deformations of a Metal Surface", J. Tech. Phys. (USSR), 25, 546.

[6] Meecham, W. C., (1956), "Variational Method for the Calculation of the Distribution of Energy Reflected from a Periodic Surface", J. Appl. Phys, 27, 361-367.

[7] Brekhovskikh, L. M. (1952), "Diffraction of Electromagnetic Waves at Uneven Surfaces", J. Exp. Theor. Phys., USSR, 23, 275-304.

[8] Senior, T. B. A. (1959), "The Scattering of Electromagnetic Waves by a Corrugated Sheet", Canad. J. Phys., 37, 781-797 and 1572.

[9] Beckmann, P. and Spizzichino, A. (1963), "The Scattering of Electromagnetic Waves from Rough Surfaces", Macmillan, New York.

reliable results been obtained [10-15]. An excellent survey of work done on scattering from periodic surfaces can be found in the report by Tong and Senior [13].

The present study uses an independent approach to the problem of scattering from periodic metal surfaces. It is basically an adaptation and extension of the physical optics approximation into a rigorous theory which applies to surfaces with radii of curvature not necessarily large compared to wavelength. The specific problem being considered is that of scattering of plane waves from a sinusoidally varying perfectly conducting surface. The current induced in this surface (which produces the scatter radiation) is assumed to have the form

$$K = K_p F$$

where K_p is the physical optics approximation of this current - or a suitably chosen modification of it - and F is a Fourier Series expansion with a fundamental period equal to the period of the surface (to satisfy Floquet's theorem for periodic structures). By imposing the so called extended boundary condition, i.e., the physical constraint that the field radiated by the induced surface currents into the lower (shielded) half-space cancels the incident plane wave in this region, a linear system is obtained for the complex Fourier coefficients of the current distribution. The matrix elements of this

---

[10] Zaki, K. A., and Neureuther, A. R. (1971), "Scattering from a Perfectly Conducting Surface with Sinusoidal Height Profile, TE Polarization" IEEE Trans., AP-19, 208-214.

[11] Zaki, K. A., and Neureuther, A. R., (1971), "Scattering from a Perfectly Conducting Surface with a Sinusoidal Height Profile, TM Polarization" IEEE Trans., AP-19, 747-751.

[12] Green, R. B., (1970), "Diffraction Efficiences for Infinite Perfectly Conducting Gratings of Arbitrary Profile," IEEE Trans., MTT-18, 313-318.

[13] Tong, T. C-H and Senior, T. B. A. (1972), "Scattering of Electromagnetic Waves by a Periodic Surface with Arbitrary Profile", University of Michigan, Dept., of Electrical and Computing Engineering, Radiation Laboratory, Scientific Report No. 13, AFCRL-72-0258.

[14] Hessel, A. and Shmoys, J., "Computer Analysis of Propagation Reflection Phenomena," Polytechnic Institute of Brooklyn Scientific Report, Contract number DAAB07-73-M-2716 August 1973.

[15] Ikuno, H. and Yasuura K., "Improved Point-Matching Method with Application to Scattering from a Periodic Surface," IEEE Transaction Antenna and Prop., AP-21, pp 657-662, September 1973.

system and the inhomogeneous terms are integrals which can be evaluated leading to closed form expressions in terms of Bessel functions. Numerical evaluation of the theory thus involves solution of a linear system. But since the coefficients of this system are closed form expressions, no time consuming integrations are involved in setting-up the coefficient matrix. Solution of this system (after appropriate truncation) then allows the determination of the scattered field in the upper (illuminated) half-space away from the conducting surface; the complex amplitudes of the plane wave spectrum of this field are series of Bessel functions. By relaxing the degree of truncation of the linear system in the numerical evaluation (i.e., by increasing the order of the system actually evaluated), it should in principle be possible to arbitrarily increase the accuracy of the approach, although this has not yet been demonstrated.

One of the reasons for conducting the present study was to investigate whether or not the specular reflection coefficient of periodically corrugated metal surfaces is significantly dependent on the direction of polarization of the incident field. Thus both TE- and TM-polarization of this field are considered where, TE-polarization denotes the case that the electric field strength is directed parallel to the direction of the surface grooves and TM-polarization the case that the magnetic field strength has this direction. Interest in this polarization question stemmed from the need to verify the superiority of TM- over TE-polarization in minimizing false guidance of microwave scanning beam landing systems [16]. Reflections from large metal structures near runways such as hangars (which usually have periodically corrugated walls and doors) or rows of parked airplanes are a likely cause of misinformation received by landing aircraft, and the question arises if such reflections can be reduced by appropriate choice of polarization. The numerical results confirm experimental evidence that TM-polarization in general leads to substantially less specular reflection than TE-polarization, in particular in the practically interesting range of low incidence angles (near grazing). At grazing incidence itself the specular amplitude reflection coefficient of course is -1, for both TE- and TM-polarization.

For a summary of the new theory we refer to a preceding shorter report [17] which also discusses the case of circular polarization of the incident field. It is shown in this report that circular polarization is highly effective in reducing higher order grazing lobes with the same polarization (cir-

---

[16] Demko, P. S., (1972), "Polarization/Multipath Study", Tech. Memorandum VL-5-72, Avionics Laboratory, U.S. Army Electronics Command Ft. Monmouth, N.J.

[17] Schwering, F. and Whitman, G., (1977) "Scattering by Sinusoidal Surfaces", Tech. Rept. ECOM-4496, Communications/ADP Laboratory, U.S. Army Electronics Command, Fort Monmouth, N.J.

cular with the same sense of rotation) as the incident wave. On the other hand, grating lobes with the opposite sense of rotation are, in general, present with significant amplitudes. Such grating lobes, however, would not be received by an airborne antenna polarized for optimum reception of the primary microwave beam.

2. FORMULATION OF THE PROBLEM

Consider a perfectly conducting sinusoidally varying surface defined by the relation

$$z_o = f(x_o) = h \sin (2\pi \frac{x_o}{d}), \; -\infty < x_o, y_o < \infty. \qquad (1)$$

A uniform plane wave is assumed incident upon this surface at an angle of θ degrees with a suppressed time dependence exp (iωt); see Fig. 1. The field strength components of the primary wave are in the case of TE-polarization

$$E_y^p = e^{ik(x \sin\theta - z \cos\theta)} \qquad (2a)$$

$$\sqrt{\frac{\mu_o}{\varepsilon_o}} H_x^p = \frac{1}{ik} \frac{\partial E_y^p}{\partial z}, \sqrt{\frac{\mu_o}{\varepsilon_o}} H_z^p = -\frac{1}{ik} \frac{\partial E_y^p}{\partial x}$$

and in the case of TM-polarization

$$H_y^p = e^{-ik(x \sin\theta - z \cos\theta)} \qquad (2b)$$

$$\sqrt{\frac{\varepsilon_o}{\mu_o}} E_x^p = -\frac{1}{ik} \frac{\partial H_y^p}{\partial z}, \sqrt{\frac{\varepsilon_o}{\mu_o}} E_z^p = \frac{1}{ik} \frac{\partial H_y^p}{\partial x}$$

In the TE-case, the electric field strength of the primary wave is assumed to have unit amplitude and in the TM-case, the magnetic field strength; $k = \frac{2\pi}{\lambda}$ is the wave number.

As it is well known from the theory of optical gratings, the incidence power is scattered into a finite number of discrete directions θ_m

$$\sin\theta_m = \sin\theta + m \frac{\lambda}{d} \qquad (3a)$$

where λ is the wavelength and m is any integer in the range

$$-\frac{d}{\lambda}(1 + \sin\theta) \le m \le \frac{d}{\lambda}(1 - \sin\theta) \qquad (3b)$$

The scatter field, in other words, comprises a finite spectrum of propagating plane waves carrying the real power reflected by the surface. This spectrum is supplemented by an infinite but discrete set of evanescent plane waves associated with the integer m values outside the range (3b). For such m-

values the propagation angles (3a) become complex, describing inhomogeneous waves exponentially decreasing in the positive z-direction. In the case of TE-polarization, the scatter field thus takes the form

$$E_y^s = \sum_{m=-\infty}^{\infty} E_m^{(1)} e^{-ik(x \sin\theta_m + z\cos\theta_m)}$$

$$\sqrt{\frac{\mu_o}{\varepsilon_o}}\, H_x^s = \frac{1}{ik}\frac{\partial E_y^s}{\partial z}, \quad \sqrt{\frac{\mu_o}{\varepsilon_o}}\, H_z^s = -\frac{1}{ik}\frac{\partial E_y^s}{\partial x} \tag{4a}$$

and in the case of TM-polarization

$$H_y^s = \sum_{m=-\infty}^{\infty} H_m^{(1)} e^{-ik(x \sin\theta_m + z\cos\theta_m)}$$

$$\sqrt{\frac{\varepsilon_o}{\mu_o}}\, E_x^s = -\frac{1}{ik}\frac{\partial H_y^s}{\partial z}, \quad \sqrt{\frac{\varepsilon_o}{\mu_o}}\, E_z^s = \frac{1}{ik}\frac{\partial H_y^s}{\partial x} \tag{4b}$$

where $E_m^{(1)}$ and $H_m^{(1)}$ are the complex amplitudes of the component plane waves of the scatter field. Using (3a) it can be seen that these component waves, except for a common factor $e^{-ikx\sin\theta}$, are periodic in x with the period d of the surface profile (Floquet's theorem). They have therefore been termed space harmonics.

3. INTEGRAL REPRESENTATION OF SCATTER FIELD AMPLITUDES

In the case of TE-polarization, the incident field induces y-directed currents in the metal surface. The electric field strength of the scatter field can be derived from this (unknown) current distribution $K_y(x_o)$ according to the relation

$$E_y^s(x,z) = -\frac{\omega\mu_o}{4}\int_{S_o} K_y(x_o)\, H_o^{(2)}(kr)\,dS_o \tag{5a}$$

where the integration is taken over the profile of the scatter surface S_o; and r is the radial distance between source and field point:

$$r = \left\{(x-x_o)^2 + (z-z_o)^2\right\}^{\frac{1}{2}}$$

The kernel of the integral i.e., the Hankel function (multiplied by - j/4) is the two-dimensional Green's function of free space. Meanwhile, for TM-polarization, the magnetic field strength of the scatter field can be derived from the current distribution in the metal surface, $K_t(x_o)$ which, in this case, flows normal to the y-direction (but, of course, tangential to the surface):

$$H_y^s(x,z) = \frac{i}{4}\int_{S_o} K_t(x_o)\,\frac{\partial}{\partial n_o}\, H_o^{(2)}(kr)\,dS_o \tag{5b}$$

where $\partial/\partial n_o$ denotes the derivative normal to the surface S_o.

Since the integrals in eqs. (5a) and (5b) extend over an infinite range, they are not amenable to numerical evaluation. However, because of the periodicity of the surface profile, these integrals can be transformed into integrals extending over one period only. Using Floquet's theorem we conclude that the surface current densities $K_y(x_o)$ and $K_t(x_o)$ are periodic with d, apart from a phase factor $\exp(-ikx_o \sin\theta)$ determined by the incident plane wave. Hence

$$K_y(x_o) = e^{-ikx_o \sin\theta}\hat{K}_y(x_o) \tag{6a}$$

$$K_t(x_o) = e^{-ikx_o \sin\theta}\hat{K}_t(x_o) \tag{6b}$$

where

$$\hat{K}_y(x_o+d) = \hat{K}_y(d) \tag{6c}$$

$$\hat{K}_t(x_o+d) = \hat{K}_t(d) \tag{6d}$$

Subdividing integrals (5a) and (5b) into sums of integrals, each extending over one period only of the scatter profile, we obtain after interchanging summation and integration

$$E_y^s(x,z) = -\frac{\omega\mu_o}{4}\int_{-d/2}^{d/2} g_E(x-x_o,z-z_o)K_y(x_o)\sqrt{1+z_o'^2}\,dx_o \tag{7a}$$

$$H_y^s(x,z) = \frac{i}{4}\int_{-d/2}^{d/2} g_H(x-x_o,z-z_o)K_t(x_o)dx_o \tag{7b}$$

where g_E and g_H are given by

$$g_E(x-x_o,z-z_o) = \sum_{n=-\infty}^{\infty} H_o^{(2)}\left[k\left\{(x-x_o-nd)^2+(z-z_o)^2\right\}^{\frac{1}{2}}\right]e^{-inkd\sin\theta} \tag{7c}$$

$$g_H(x-x_o\ ,\ z-z_o) = \left\{\left(z_o'\frac{\partial}{\partial x_o}-\frac{\partial}{\partial z_o}\right)\sum_{n=-\infty}^{\infty} H_o^{(2)}\left[k\left\{(x-x_o-nd)^2 + (z-z_o)^2\right\}^{\frac{1}{2}}\right]\right\}e^{-inkd\sin\theta} \tag{7d}$$

We have expressed the surface element ds_o and the differential $\partial/\partial n_o$ (in the expression for g_H in terms of x_o and z_o:

$$ds_o = \sqrt{1 + z_o'^2}\, dx_o$$

$$\frac{\partial}{\partial n_o} = \frac{1}{\sqrt{1 + z_o'^2}} \left(z_o' \frac{\partial}{\partial x_o} - \frac{\partial}{\partial z_o}\right)$$

where z_o and z_o' are the height and slope respectively of the surface profile. Hence, with eq (1).

$$z_o' = \frac{dz_o}{dx_o} = 2\pi \frac{h}{d} \cos\left(2\pi \frac{x_o}{d}\right) \tag{8}$$

We now apply Poisson's summation formula [18], [19] which we write in a form appropriate for our present purpose:

$$\sum_{m=-\infty}^{\infty} H_o^{(2)}[k\{(x-x_o-md)^2 + (z-z_o)^2\}^{\frac{1}{2}}] e^{-imkd\sin\theta} = \frac{2}{kd} \sum_{m=-\infty}^{\infty} \frac{1}{\cos\theta_m} e^{-ik[(x-x_o)\sin\theta_m + |z-z_o|\cos\theta_m]} \tag{9}$$

with

$$\sin\theta_m = \sin\theta + m\,\lambda/d \tag{9a}$$

and

$$\mathrm{Re}\,(\cos\theta_m) > 0,\ \mathrm{Im}(\cos\theta_m) < 0 \tag{9b}$$

Then equations (7a) to (7d) reduce-after interchanging integration and summation a second time-to space harmonics representations in accordance with eqs. (4a) and (4b). In the case of TE-polarization we obtain

$$E_y^s = \sum_{m=-\infty}^{\infty} E_m^{(1)}\, e^{-ik(x\sin\theta_m + z\cos\theta_m)} \tag{10a}$$

---

[18] Felsen, L. B. and Marcuvitz, N. (1973), "Radiation and Scattering of Waves", Prentice-Hall, New Jersey.

[19] Morse, P. M., and Feshbach, H. (1953), "Methods of Theoretical Physics-Part I," McGraw-Hill, New York.

with

$$E_m^{(1)} = -\frac{1}{2d}\sqrt{\frac{\mu_o}{\varepsilon_o}}\,\frac{1}{\cos\theta_m}\int_{-d/2}^{d/2} K_y(x_o)e^{ik(x_o\sin\theta_m + z_o\cos\theta_m)}\sqrt{1+z_o'^2}\,dx_o \tag{11a}$$

and in the case of TM-polarization

$$H_y^s = \sum_{m=-\infty}^{\infty} H_m^{(1)}\,e^{-ik(x\sin\theta_m + z\cos\theta_m)} \tag{10b}$$

with

$$H_m^{(1)} = \frac{1}{2d}\int_{-d/2}^{d/2} K_t(x_o)e^{ik(x_o\sin\theta_m + z_o\cos\theta_m)}\,[1-z_o'\tan\theta_m]\,dx_o \tag{11b}$$

In both cases we assume $z \geq h = \mathrm{Max}\,(z_o)$ to insure convergence, and

$$\mathrm{Re}(\cos\theta_m) > 0, \quad \mathrm{Im}(\cos\theta_m) < 0$$

to satisfy the radiation condition (see eq. (9b)).

Note that while eqs. (7a) to (7d) represent the scatter field at all points above the scatter surface, including the space range within the grooves, the space harmonics representation (10a) and (10b) converge in the space range $z \geq h$ but may diverge within the grooves. Hence this representation cannot, in general, be used to formulate the boundary condition $E_{tang} = 0$ at the metal surface (Rayleigh Hypothesis [20]). For our present purpose the space harmonics method is appropriate: the scatter problem is solved using an approach which does not require explicite formulation of this conventional boundary condition.

4. CALCULATION OF INDUCED SURFACE CURRENT DENSITIES

Equations (10a) to (11b) provide a representation of the scatter field of the periodic surface in terms of the (unknown) current distribution induced by the incident plane wave. To determine this current distribution we use the following approach. The metal surface is assumed removed while the current distribution existing in this surface is maintained in place. If an approximation is used for this current distribution, deviating from the actual current density, then a field will exist not only in the half-space above but also in the half-space below the current carrying surface. This latter field can be derived by a procedure similar to that employed in calculating the field in the upper half-space; the condition that this field must be identically zero is then utilized to determine the correct current distribution on the metallic surface.

Equations (7a) to (7d) hold for $z > 0$ as well as for $z < 0$. Inserting

---

[20] For a discussion of Rayleigh's method and it's limitations see Tong and Senior [13], or Millar, R. F., "Some Controversial Aspects of Scattering by Periodic Structures, XVII General Assembly of URSI, Session VI-5", Warsaw, August, 1972.

Poissons summation formula (9), while observing that in the lower half-space $z < z_o$, we obtain in the case of TE-polarization

$$E_y^T = \sum_{m=-\infty}^{\infty} E_m^{(2)} \; e^{-ik(x \sin\theta_m - z \cos\theta_m)} \tag{12a}$$

$$E_m^{(2)} = -\frac{1}{2d}\sqrt{\frac{\mu_o}{\varepsilon_o}}\,\frac{1}{\cos\theta_m}\int_{-d/2}^{d/2} K_y(x_o)\, e^{ik(x_o \sin\theta_m - z_o \cos\theta_m)}\sqrt{1+z_o'^2}\,dx_o \tag{13a}$$

and in the case of TM-polarization

$$H_y^T = \sum_{m=-\infty}^{\infty} H_m^{(2)} \; e^{-ik(x \sin\theta_m - z \cos\theta_m)} \tag{12b}$$

$$H_m^{(2)} = -\frac{1}{2d}\int_{-d/2}^{d/2} K_t(x_o)\, e^{ik(x_o \sin\theta_m - z_o \cos\theta_m)}(1+z_o' \tan\theta_m)\,dx_o \tag{13b}$$

As before, the propagation angles θ_m are given by eqs. (9a) and (9b). To insure convergence it is assumed that $z \leq -h = \mathrm{Min}\,(z_o)$; for z_o and z_o' see eqs. (1) and (8), respectively. In the lower as in the upper half-space the scatter field is thus obtained as a superposition of space harmonics i.e., as a discrete spatial spectrum of propagating and evanescent plane waves whose x-dependence-except for a common phase facter-is periodic with the period d of the surface profile. The propagation directions θ_m of the upper and lower half-spaces correspond to each other; they are related by imaging at the plane $z = 0$.

The current distribution existing on a metal surface is now obtained from the condition that the field in the lower half-space must be identically zero. This means that the space harmonic of order zero must cancel the incident plane wave (it travels in the same direction) and that all higher order space harmonics must vanish. Hence for TE-polarization

$$E_m^{(2)} = \begin{cases} 0 & \text{for } m \neq 0 \\ -1 & \text{for } m = 0 \end{cases} \tag{14a}$$

and for TM - polarization

$$H_m^{(2)} = \begin{cases} 0 & \text{for } m \neq 0 \\ -1 & \text{for } m = 0 \end{cases} \tag{14b}$$

These equations in conjunction with eqs. (13a) and (13b) form a set of integral relations for the current distributions K_y and K_t. To solve these equations it is expedient to write K_y and K_t in the following form:

$$K_y(x_o) = \sqrt{\frac{\varepsilon_o}{\mu_o}}\, K_y'(x_o\ )$$

$$= 2\sqrt{\frac{\varepsilon_o}{\mu_o}}\ \frac{\cos\theta}{\sqrt{1+z_o'^2}}\, e^{-ik(x_o \sin\theta - z_o \cos\theta)} F_y(x_o) \tag{15a}$$

$$K_t(x_o) = -\,2\, e^{-ik(x_o \sin\theta - z_o \cos\theta)} F_t(x_o), \tag{15b}$$

where F_y and F_t are Fourier series with a fundamental period equal to the period of the surface profile [21]:

$$F_{y,t}(x_o) = \sum_{n=-\infty}^{\infty} C_n^{E,H}\, e^{i2\pi n \frac{x_o}{d}} \tag{15c}$$

Note that expressions (15a) and (15b) satisfy Floquet's theorem as expressed by eqs. (6a) and (6b) and observe, furthermore, that a functional dependence closely resembling the physical optics approximation has been chosen. The expression for K_t, eq. (15b), becomes identical with this approximation if F_t is replaced by unity (first order approximation). If F_y is replaced by unity, expression (15a) for K_y reduces to a modified version [22] of the physical optics approximation; however, it has been found that this modification enhances computational accuracy. Furthermore, it has been shown that an integral equation for K_y, obtained by formulating the boundary condition $E_{tang} = 0$ at the metal surface, will yield exactly this modified approximation when evaluated in the vicinity of the singularities of its kernel [23]

---

[21] The factor $\sqrt{\varepsilon_o/\mu_o}$ appearing in the expression for K_y but not in that for K_t is due to the normalization used: unit amplitude of the electric field strength of the primary wave in the TE-case and of the magnetic field strength in the TM-case.

[22] The physical optics approximation would result if the factor $\cos\theta$ in front of the exponential function (phase term) in eq. (15a) were replaced by $\cos\theta + z_o' \sin\theta$.

[23] See reference [13] on page 2.

Inserting eqs. (15a), (15b) and (15c) into eqs. (13a) and (13b) and performing the integrations leads to closed form expressions in terms of Bessel functions. Thus,

$$E_m^{(2)} = -\frac{1}{d}\frac{\cos\theta}{\cos\theta_m}\int_{-d/2}^{d/2}\sum_{n=-\infty}^{\infty} C_n^E\, e^{i2\pi\left[(m+n)\frac{x_o}{d} + \frac{h}{\lambda}\sin(2\pi\frac{x_o}{d})(\cos\theta-\cos\theta_m)\right]}\, dx_o \tag{16a}$$

$$= \frac{\cos\theta}{\cos\theta_m}\sum_{n=-\infty}^{\infty}(-1)^{n+m+1} J_{m+n}(\alpha_{2m})\, C_n^E$$

and

$$H_m^{(2)} = \frac{1}{d}\int_{-d/2}^{d/2}\sum_{n=-\infty}^{\infty} C_n^H\, e^{i2\pi\left[(m+n)\frac{x_o}{d} + \frac{h}{\lambda}\sin(2\pi\frac{x_o}{d})(\cos\theta-\cos\theta_m)\right]}\cdot$$

$$\cdot\left[1+2\pi\frac{h}{d}\cos(\frac{2\pi x_o}{d})\tan\theta_m\right]dx_o$$

$$= \sum_{n=-\infty}^{\infty}(-1)^{m+n}\left[J_{m+n}(\alpha_{2m}) - \frac{p_m}{2}(J_{m+n+1}(\alpha_{2m}) + J_{m+n-1}(\alpha_{2m}))\right] C_n^H \tag{16b}$$

$$= \sum_{n=-\infty}^{\infty}(-1)^{m+n}\left[1-(n+m)\frac{p_m}{\alpha_{2m}}\right] J_{m+n}(\alpha_{2m})\, C_n^H ,$$

with

$$\alpha_{2m} = \frac{2\pi h}{\lambda}(\cos\theta-\cos\theta_m),\quad p_m = \frac{2\pi h}{d}\tan\theta_m ,$$

where the angles θ_m are defined by eqs. (9a) and (9b). With these expressions for $E_m^{(2)}$ and $H_m^{(2)}$, conditions (14a) and (14b) reduce to the following linear systems for the unknown current coefficients C_n^E and C_n^H:

$$\sum_{n=-\infty}^{\infty}(-1)^n J_{n+m}(\alpha_{2m})\, C_n^E = 0 \qquad \text{for } m \neq 0 \tag{17a}$$

$$\sum_{n=-\infty}^{\infty}(-1)^{n+1} J_n(0)\, C_n^E \equiv -C_o^E = -1 \qquad \text{for } m = 0 \tag{17b}$$

and

$$\sum_{n=-\infty}^{\infty}(-1)^n\left[1-(n+m)\frac{p_m}{\alpha_{2m}}\right] J_{m+n}(\alpha_{2m})C_n^H = 0 \text{ for } m \neq 0 \tag{17c}$$

$$\sum_{n=-\infty}^{\infty}(-1)^n\left[J_o(0) - \frac{p_o}{2}(J_{n+1}(0) + J_{n-1}(0))\right]C_n^H \equiv C_o^H + \frac{p_o}{2}(C_{+1}^H + C_{-1}^H) = -1$$

$$\text{for } m = 0 \tag{17d}$$

Eqs. (17b) and (17d) yield explicit expressions for the zero order coefficients C_o^E and C_o^H, respectively:

$$C_o^E = 1 \tag{18a}$$

$$C_o^H = -1 - \frac{p_o}{2}(C_{+1}^H + C_{-1}^H) \tag{18b}$$

Using these expressions in eqs. (17a) and (17c), the linear systems for the higher order current coefficients take the form:

$$\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} A_{mn}\, C_n^E = -A_{mo},\; m = \pm 1, \pm 2, \ldots, \pm\infty$$

$$A_{mn} = (-1)^n J_{m+n}(\alpha_{2m}) \tag{18c}$$

$$\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \left[B_{mn} - \delta_{\pm}\frac{p_o}{2}B_{mo}\right]C_n^H = B_{mo},\; m = \pm 1, \pm 2, \ldots, \pm\infty, \tag{18d}$$

$$B_{mn} = (-1)^n\left[1-(m+n)\frac{p_m}{\alpha_{2m}}\, J_{m+n}(\alpha_{2m})\right]$$

$$\delta_{\pm} = \begin{cases} 1 \text{ for } n = \pm 1 \\ 0 \text{ for } n \neq \pm 1 \end{cases}$$

Solution of the linear systems (18c) and (18d) requires a matrix inversion which is efficiently handled by available computer routines. A particular advantage is that all matrix coefficients and inhomogeneous terms are closed form expressions in well-known functions so that no time consuming numerical integrations are involved in computing the coefficients of the matrix to be inverted, though a routine for Bessel functions of complex arguments is needed. Inversion of course requires truncation of the in principle infinite linear systems. We assume without proof that the current distributions computed will converge towards the actual distributions when the degree of truncation is relaxed, i.e., when the order of the linear systems is step by step increased. As the critique of the Rayleigh method has shown such assumptions have to be regarded with caution. In contrast to the Rayleigh hypothesis, there are no obvious physical reasons for doubting the validity of the present approach; but problems have been encountered in the numerical evaluation when the groove depth (2h) exceeded 1 to 2 wavelengths.

(See part II of this report).

5. SERIES REPRESENTATION OF SCATTER FIELD AMPLITUDES

When the Fourier coefficients C_n^E and C_n^H, of the current distributions K_y and K_t are known, evaluation of the plane wave amplitudes of the scatter field in the upper half-space is straight forward. In the case of TE-polarization we obtain by insertings eqs. (15a) and (15c) into eq. (11a)

$$E_m^{(1)} = -\frac{1}{d}\frac{\cos\theta}{\cos\theta_m}\int_{-d/2}^{d/2}\sum_{n=-\infty}^{\infty} C_n^E\, e^{i2\pi[(m+n)\frac{x_o}{d}+\frac{h}{\lambda}\sin(2\pi\frac{x_o}{d})(\cos\theta+\cos\theta_m)]}\, dx_o \tag{19a}$$

$$= \frac{\cos\theta}{\cos\theta_m}\sum_{n=-\infty}^{\infty}(-1)^{m+n+1}\, J_{m+n}(\alpha_{1m})C_n^E$$

and in the case of TM-polarization by using eqs. (15b) and (15c) in eq. (11b)

$$H_m^{(1)} = -\frac{1}{d}\int_{-d/2}^{d/2}\sum_{n=-\infty}^{\infty} C_n^H\, e^{i2\pi[(m+n)\frac{x_o}{d}+\frac{h}{\lambda}\sin(2\pi\frac{x_o}{d})(\cos\theta+\cos\theta_m)]} \tag{19b}$$

$$\cdot(1-2\pi\frac{h}{d}\cos(\frac{2\pi x_o}{d})\tan\theta_m)\, dx_o$$

$$= \sum_{n=-\infty}^{\infty}(-1)^{m+n+1}[1+(m+n)\frac{p_m}{\alpha_{1m}}]\, J_{m+n}(\alpha_{1m})\, C_n^H \tag{19c}$$

where *in both cases*

$$\alpha_{1m} = 2\pi\frac{h}{\lambda}(\cos\theta+\cos\theta_m)\,,\quad p_m = 2\pi\frac{h}{d}\tan\theta_m \tag{19c}$$

We have used here eqs. (1) and (8) for z_o and z'_o, eq. (9a) for $\sin\theta_m$, and have furthermore assumed that the Fourier series of the current distributions converge sufficiently to permit interchanging summation and integration.

6. CONSERVATION OF POWER DENSITY AND RECIPROCITY THEOREM

Two physical criteria are used to check the validity of our numerical results. They are conservation of power and reciprocity. The former requires that the power reflected per unit area from a perfectly conducting surface equal the power incident per unit area upon the same surface, i.e.,

$$S_{\perp}^{inc} = S_{\perp}^{refl}$$

where

$$S_{\perp} = \frac{1}{2}\,\mathrm{Re}(\underline{E} \times \underline{H}^{*}\cdot\underline{n})$$

is the normal component of the Poynting vector and $\underline{n}$ is a unit vector perpendicular to the surface. In the present case it is convenient to formulate this condition for one period of the metal surface, utilizing a plane horizontal test surface subtended between two adjacent maxima of the surface profile (to insure convergence of the space harmonics representation of the scatter field). For the primary power incident through the test surface we obtain with eqs. (2a) and (2b)

$$P^{inc} = \begin{cases} \frac{d}{2}\sqrt{\frac{\varepsilon_o}{\mu_o}}\cos\theta & \text{for TE-polarization} \quad (20a) \\ \frac{d}{2}\sqrt{\frac{\mu_o}{\varepsilon_o}}\cos\theta & \text{for TM-polarization} \end{cases}$$

and for the reflected power propagating through this surface we obtain by using eqs. (4a) and (4b)

$$P^{refl} = \begin{cases} \frac{d}{2}\sqrt{\frac{\varepsilon_o}{\mu_o}}\sum_m |E_m^{(1)}|^2\cos\theta_m & \text{for TE-polarization} \quad (20b) \\ \frac{d}{2}\sqrt{\frac{\mu_o}{\varepsilon_o}}\sum_m |H_m^{(1)}|^2\cos\theta_m & \text{for TM-polarization} \end{cases}$$

where the summation is limited to the propagating space harmonics i.e., to spectral orders m satisfying condition (3b). Eq. (20b) expresses the fact that the space harmonics are mutually orthogonal over one period. Conservation of power thus requires

$$\cos\theta = \begin{cases} \sum_m |E_m^{(1)}|^2\cos\theta_m & \text{for TE-polarization} \quad (20c) \\ \sum_m |H_m^{(1)}|^2\cos\theta_m & \text{for TM-polarization} \quad (20d) \end{cases}$$

providing a criterion for checking the accuracy of the computed results.

The second physical condition tested is reciprocity. As shown in appendix A, the Lorentz reciprocity theorem applied to the problem of periodic surface scattering may be stated as follows: If a plane wave incident from direction θ^a produces a space harmonic of order m propagating in the direction $\theta_m^a = -\theta^b$, then a plane wave arriving from direction θ^b produces a space harmonic of the same order m traveling in direction $\theta_m^b = -\theta^a$

[24] and the complex amplitudes of the two space harmonics are related by

$$(E_m^{(1)})^a \cos\theta_m^a = (E_m^{(1)})^b \cos\theta_m^b \tag{21a}$$

for TE-polarization and by

$$(H_m^{(1)})^a \cos\theta_m^a = (H_m^{(1)})^b \cos\theta_m^b \tag{21b}$$

for TM-polarization. We have assumed here that the two incident waves have equal amplitude and phase.

The power and reciprocity criteria supplement each other. While the former criterion checks on the <u>accumulated</u> <u>power</u> of <u>all</u> propagating space harmonics, the reciprocity criterion tests the <u>amplitude</u> and <u>phase</u> of individual space harmonics.

APPENDIX A

Reciprocity Relation for Periodic Surface Scattering

Two TE-polarized plane waves, with propagation angles θ^a and θ^b,

$$\underline{E}_{inc}^a = e^{-ik(x \sin\theta^a - z \cos\theta^a)} \underline{e}_y \tag{A.1}$$

$$\underline{H}_{inc}^a = \sqrt{\frac{\varepsilon_o}{\mu_o}}\, e^{-ik(x \sin\theta^a - z \cos\theta^a)} (\underline{e}_x \cos\theta^a + \underline{e}_z \sin\theta^a)$$

and

$$\underline{E}_{inc}^b = e^{-ik(x \sin\theta^b - z\cos\theta^b)} \underline{e}_y \tag{A.2}$$

$$\underline{H}_{inc}^b = \sqrt{\frac{\varepsilon_o}{\mu_o}}\, e^{-ik(x \sin\theta^b - z\cos\theta^b)} (\underline{e}_x \cos\theta^b + \underline{e}_z \sin\theta^b)$$

---

[24] The minus signs in the relations $\theta_m^a = -\theta^b$ and $\theta_m^b = -\theta^a$ are a matter of definition. Propagation angles of incident and scattered waves (ingoing and outgoing waves) are counted with opposite signs; see Fig. 1.

are assumed incident on a periodic metal surface with height profile (1), i.e.,

$$z_o = h \sin (2\pi \frac{x_o}{d}) \quad -\infty < x_o < +\infty \tag{A.3}$$

The respective scatter fields are denoted by $\underline{E}^a_{sc}$, $\underline{H}^a_{sc}$ and $\underline{E}^b_{sc}$, $\underline{H}^b_{sc}$. In the space range above the surface grooves, $z > \text{Max}\ (z_o) = h$, these scatter fields can be expressed by their space harmonic representations:

$$\underline{E}^a_{sc} = \sum_{m=-\infty}^{\infty} E^a_m e^{-ik(x \sin\theta^a_m + z\cos\theta^a_m)} \underline{e}_y \tag{A.4}$$

$$\underline{H}^a_{sc} = \sqrt{\frac{\epsilon_o}{\mu_o}} \sum_{m=-\infty}^{\infty} E^a_m e^{-ik(x \sin\theta^a_m + z\cos\theta^a_m)} (-\underline{e}_x \cos\theta^a_m + \underline{e}_z \sin\theta^a_m)$$

and

$$\underline{E}^b_{sc} = \sum_{m=-\infty}^{\infty} E^b_m e^{-ik(x \sin\theta^b_m + z \cos\theta^b_m)} \underline{e}_y \tag{A.5}$$

$$\underline{H}^b_{sc} = \sqrt{\frac{\epsilon_o}{\mu_o}} \sum_{m=-\infty}^{\infty} E^b_m e^{-ik(x \sin\theta^b_m + z \cos\theta^b_m)} (-\underline{e}_x \cos\theta^b_m + \underline{e}_z \sin\theta^b_m)$$

where $\underline{e}_x$, $\underline{e}_y$, $\underline{e}_z$ are the unit vectors of the x,y,z -directions, respectively, and from eq. (9a)

$$\sin\theta^a_m = \sin\theta^a + m \frac{\lambda}{d} \tag{A.6}$$

$$\sin\theta^b_m = \sin\theta^b + m \frac{\lambda}{d} \qquad \text{for } -\infty \leq m \leq \infty.$$

The Lorentz reciprocity theorem applied to the two dimensional problem considered have states that

$$RI \equiv \oint_S (\underline{E}^a \times \underline{H}^b - \underline{E}^b \times \underline{H}^a) \cdot \underline{n} dS = 0 \ , \tag{A.7}$$

where $\underline{E}^a$, $\underline{H}^a$ and $\underline{E}^b$, $\underline{H}^b$ are arbitrary electromagnetic fields, S is any closed contour not containing (active) sources, and $\underline{n}$ is a unit vector normal to S. We chose

$$\underline{E}^a = \underline{E}^a_{inc} + \underline{E}^a_{sc}, \quad \underline{H}^a = \underline{H}^a_{inc} + \underline{H}^a_{sc}$$

$$\underline{E}^b = \underline{E}^a_{inc} + \underline{E}^a_{sc}, \quad \underline{H}^b = \underline{H}^b_{inc} + \underline{H}^b_{sc}$$

and apply the reciprocity relation to the contour

$$S = S_1 + S_2 + S_3 + S_4$$

shown in Fig. A.2 where S_2 has the length d of one period of the metal surface, S_1 and S_3 have arbitrary height, and S_4 follows closely the metal surface. Clearly, the contribution of S_4 to the reciprocity integral RI is zero since at the metal surface $\underline{E}^a$, $\underline{E}^b = 0$. On the remaining portion of the contour the scatter fields may be expressed by their space harmonics representations (A.4) and (A.5).

We now make the assumption that the direction of arrival of the incident field $\underline{E}^b_{inc}$, $\underline{H}^b_{inc}$ coincides with the propagation direction of one of the space harmonics (spectral order m = m') of the scatter field $\underline{E}^a_{sc}$, $\underline{H}^a_{sc}$. In other words, [25]

$$-\theta^b = \theta^a_{m'} \tag{A.8}$$

Then, as can be shown from eqs. (A.6), the propagation direction of one of the space harmonics of the scatters field $\underline{E}^b_{sc}$, $\underline{H}^b_{sc}$ will by necessity coincide with the direction of arrival of incident field $\underline{E}^a_{inc}$, $\underline{H}^a_{inc}$:

$$-\theta^a = \theta^b_{m'}, \tag{A.9}$$

The order of this space harmonic, m = m', is the same as in eq. (A.8) and from (A.6) and (A.8)

$$m' = -\frac{d}{\lambda}(\sin\theta^a + \sin\theta^b) \tag{A.10}$$

With a later application in mind we note that if eqs. (A.8) and (A.9) are satisfied, the propagation directions of scatter field (b) are those of scatter field (a) imaged about the z - axis, and vice versa. This statement is verified by adding eqs. (A.6) and utilizing eq. (A.10) to show that

$$\sin\theta^a_m + \sin\theta^b_n = (m+n-m')\frac{\lambda}{d} \tag{A.11}$$

from where it follows that

$$\theta^b_{m'-m} = -\theta^a_m . \tag{A.12}$$

Using eqs. (A.1), (A.2), (A.4) and (A.5) we now formulate the reciprocity integral (A.7) :

---

[25] With regard to the minus signs in eqs. (A.8) and (A.9) see Fig. 1; angles of arrival (θ^a,θ^b) and scatter angles (θ^a_m, θ^b_m) are counted with opposite signs.

$$RI = \sqrt{\frac{\epsilon_o}{\mu_o}} \int_{S_1+S_2+S_3} \Big\{ e^{i2\pi[m' \frac{x}{d} + \frac{z}{\lambda}(\cos\theta^a + \cos\theta^b)]} [-\underline{e}_x(\sin\theta^a - \sin\theta^b) + \underline{e}_z(\cos\theta^a - \cos\theta^b)]$$

$$+ \sum_{m=-\infty}^{\infty} E_m^b \, e^{i2\pi[(m'-m)\frac{x}{d} + \frac{z}{\lambda}(\cos\theta^a - \cos\theta_m^b)]} [-\underline{e}_x(\sin\theta^a - \sin\theta_m^b) + \underline{e}_z(\cos\theta^a + \cos\theta_m^b)]$$

$$+ \sum_{m=-\infty}^{\infty} E_m^a \, e^{i2\pi[(m'-m)\frac{x}{d} + \frac{z}{\lambda}(\cos\theta^b - \cos\theta_m^a)]} [+\underline{e}_x(\sin\theta^b - \sin\theta_m^a) - \underline{e}_z(\cos\theta^b + \cos\theta_m^a)]$$

$$+ \sum_{m,n=-\infty}^{\infty} E_m^a E_n^b \, e^{i2\pi[(m'-m-n)\frac{x}{d} - \frac{z}{\lambda}(\cos\theta_m^a + \cos\theta_n^b)]} [-\underline{e}_x(\sin\theta_m^a - \sin\theta_n^b) - \underline{e}_z(\cos\theta_m^a - \cos\theta_n^b)\Big\}$$

$$\cdot \underline{n} dS \qquad \text{(A.13)}$$

We have utilized here eqs. (A.6), (A.10) and (A.11) to write the exponents of the integrand in a form which shows that the integrand is periodic in x with the period d of the metal surface. As a consequence of this periodicity, the contributions of S_1 and S_3 to RI cancel each other (observe that $\underline{n} = -\underline{e}_x$ on S_1 and $\underline{n} = +\underline{e}_x$ on S_3). Furthermore, only those terms of the integrand whose x-dependent phase factors reduce to unity yield a nonzero contribution when integrated along S_2.

Thus:

$$RI = \sqrt{\frac{\epsilon_o}{\mu_o}} d \Big\{ 2E_{m'}^b \cos\theta_{m'}^b - 2E_{m'}^a \cos\theta_{m'}^a \sum_{m=-\infty}^{\infty} E_m^a E_{m'-m}^b e^{-ikz(\cos\theta_m^a + \cos\theta_{m'-m}^b)} \cdot (\cos\theta_m^a - \cos\theta_{m'-m}^b) \Big\}$$

The last term within the parenthesis (sum term) vanishes because of eq. (A.12). The reciprocity theorem (A.7) thus reduces to the condition

$$E_{m'}^a \cos\theta_{m'}^a = E_{m'}^b \cos\theta_{m'}^b \qquad \text{(A.15)}$$

This result may be stated in the following fashion. If a plane wave incident from direction θ^a produces a space harmonic traveling in direction $-\theta^b$, then a plane wave incident form direction θ^b will generate a space harmonic of the same order propagating in direction $-\theta^a$. If the two incident waves moreover have the same amplitude and phase then the two space harmonics considered also have equal phases while the ratio of their amplitudes is equal to the

inverse ratio of their direction cosines.

Application of the reciprocity theorem to TM-polarized fields follows the same pattern as used here for the case of TE-polarization. The resulting relation

$$H^a_{m'} \cos\theta^a_{m'} = H^b_{m'} \cos\theta^b_{m'} \tag{A.16}$$

corresponds in all respects to condition (A.15). $H^a_{m'}$ and $H^b_{m'}$ are the complex amplitudes of space harmonics propagating in directions $-\theta^a_m = \theta^b$ and $-\theta^b_m = \theta^a$, respectively, generated by incident plane waves of unit amplitude, zero phase and respective angles of arrival θ^a and θ^b.

APPENDIX B

Physical Optics Approximation: Summary of Formulas

With regard to the coordinates and surface parameters we refer to Fig. 1 of the main text. If the groove width of the periodic surface is sufficiently large, $d >> \lambda$, then it is reasonable to assume that the current density in this surface is primarily determined by local effects. In other words, it may be assumed that the current in each surface element is the same as if this surface element were part of an infinite metal plane (tangent plane) having the same slope as the surface element. Then for TE-polarization

$$K_y(x_o) \approx 2\,\frac{H^P_x(x_o, z_o) + z'_o\, H^P_z(x_o, z_o)}{\sqrt{1+z'^2_o}} \tag{B.1}$$

$$= 2\sqrt{\frac{\varepsilon_o}{\mu_o}}\,\frac{\cos\theta + z'_o \sin\theta}{\sqrt{1+z'^2_o}}\, e^{-ik(x_o \sin\theta - z_o \cos\theta)}$$

and for TM-polarization

$$K_t(x_o) \approx -2H^P_y(x_o, z_o) = -2e^{-ik(x_o \sin\theta - z_o \cos\theta)}, \tag{B.2}$$

where H^P_x, H^P_y, H^P_z evaluated at x_o, z_o are the magnetic field strength components of the incident plane wave at the metal surface, θ is the angle of incidence and z_o and z'_o are the height and slope of the surface profile as given by eqs. (1) and (8):

$$z_o = h \sin(2\pi \frac{x_o}{d})$$

$$z'_o = 2\pi \frac{h}{d} \cos(2\pi \frac{x_o}{d})$$

In the TE-case the electric field strength and in the TM-case the magnetic field strength of the incident plane waves have unit amplitudes.

The complex amplitudes of the space harmonics of the scatter field are obtained by using current distributions (B.1) and (B.2) in eqs. (11a) and (11b) respectively. Thus for TE-polarization

$$E_m^{(1)} = -\frac{1}{d}\frac{\cos\theta}{\cos\theta_m}\int_{x_1}^{x_2} e^{i(2\pi m \frac{x_o}{d} + kz_o(\cos\theta + \cos\theta_m)}(1+z_o' \tan\theta)\, dx_o \qquad (B.3)$$

and for TM-polarization

$$H_m^{(1)} = -\frac{1}{d}\int_{x_1}^{x_2} e^{i(2\pi m \frac{x_o}{d} + kz_o(\cos\theta + \cos\theta_m)}(1-z_o' \tan\theta_m)\, dx_o \qquad (B.4)$$

where θ_m is defined by eq. (9a). If shadow effects are taken into account, the integration is limited to the subrange in which the metal surface is directly illuminated by the incident plane wave while in the geometrical shadow zones the surface currents are assumed to be zero; See Fig. B.1. The limits of the range of integration in this case are determined by the relations:

$$x_1 + h\tan\theta \sin(2\pi \frac{x_1}{d}) = h\sqrt{\tan^2\theta - (\frac{1}{2\pi}\frac{d}{h})^2} + (x_2 - d)$$

$$x_2 = \frac{d}{2}\left\{1 - \frac{1}{\pi}\cos^{-1}(\frac{1}{2\pi}\frac{d}{h}\,\mathrm{ctg}\,\theta)\right\} \qquad (B.5)$$

If shadows effects are neglected or no geometrical shadowing occurs (i.e., for $\theta < \tan^{-1}(\frac{1}{2\pi}\frac{d}{h})$) the integration is performed over a full period of the surface profile i.e., over the range $-d/2 \leq x_o \leq d/2$. In this case the integrals (B.3) and (B.4) can be evaluated in closed form leading to identical expressions for TE-and TM-polarization:

$$E_m^{(1)} = H_m^{(1)} = (-1)^m[1 + \frac{mp_m}{\alpha_{1m}}]\, J_m(\alpha_{1m}) \qquad (B.6)$$

$$= (-1)^m \frac{1 + \cos(\theta + \theta_m)}{(\cos\theta + \cos\theta_m)\cos\theta_m} J_m(\alpha_{1m})$$

with $p_m = 2\pi \frac{h}{d}\tan\theta_m$

$$\alpha_{1m} = 2\pi\frac{h}{d}(\cos\theta + \cos\theta_m)$$

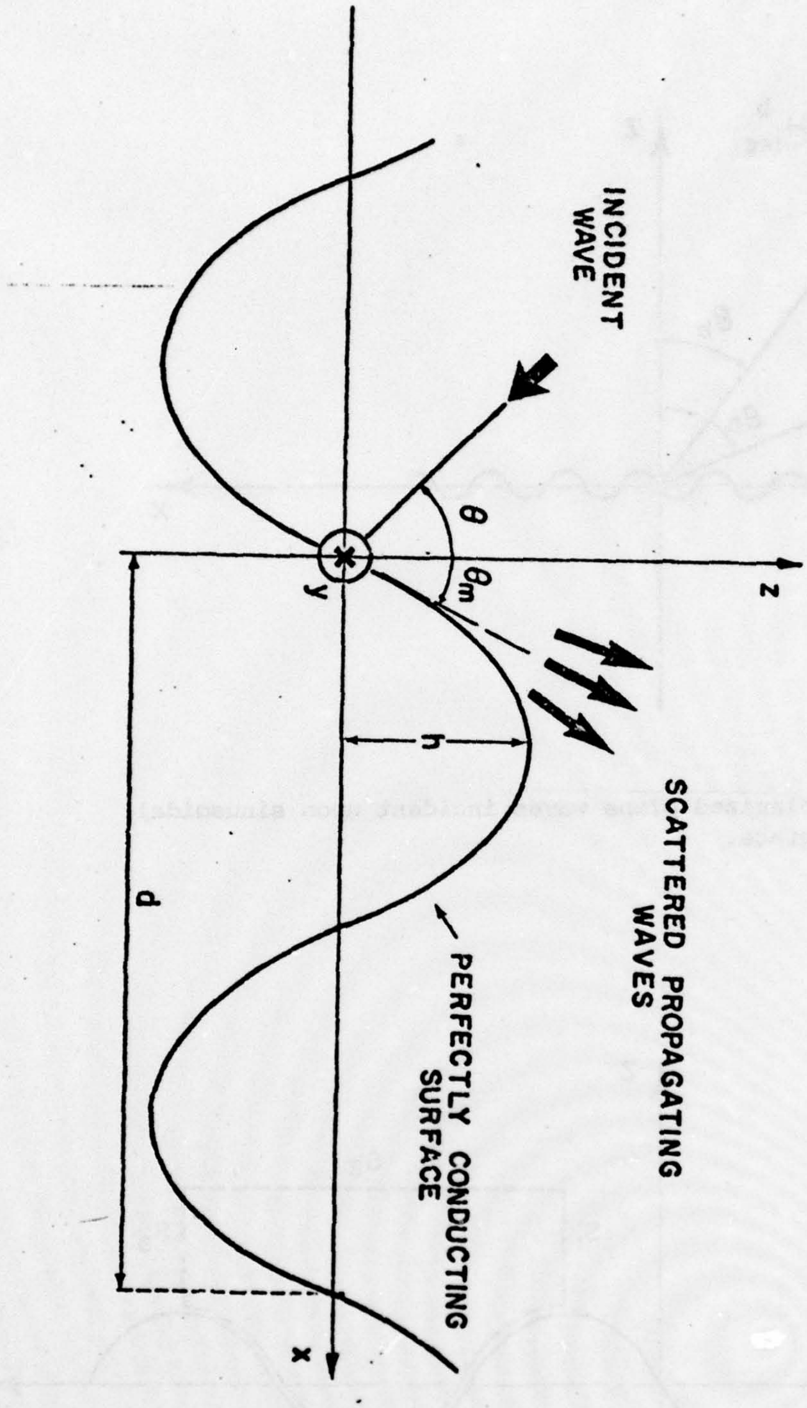


Fig. 1. Sinusoidal metal surface illuminated by plane wave. Coordinates and geometrical parameters used in analysis of scatter problem.

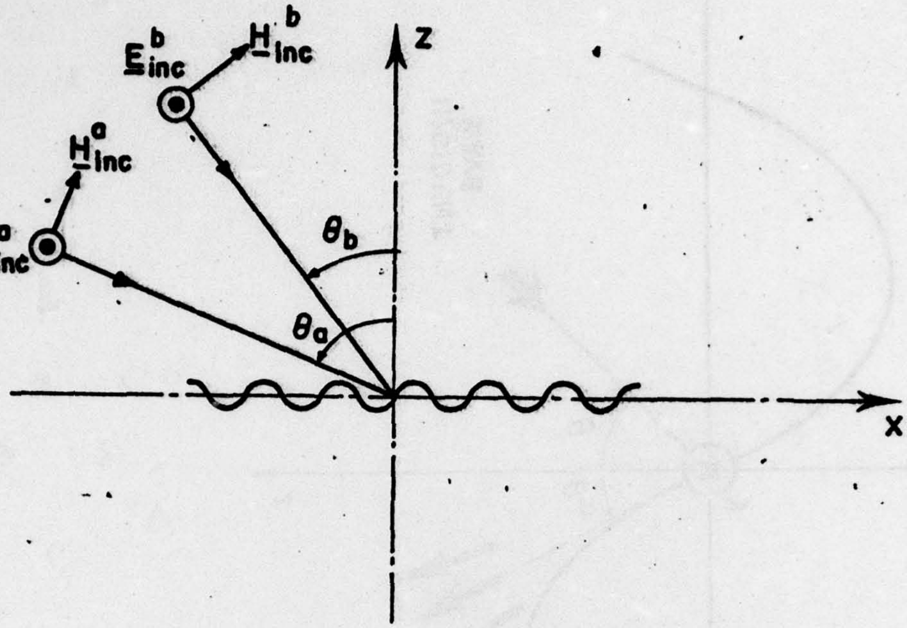


Fig. A1. Two TE-polarized plane waves incident upon sinusoidal metal surface.

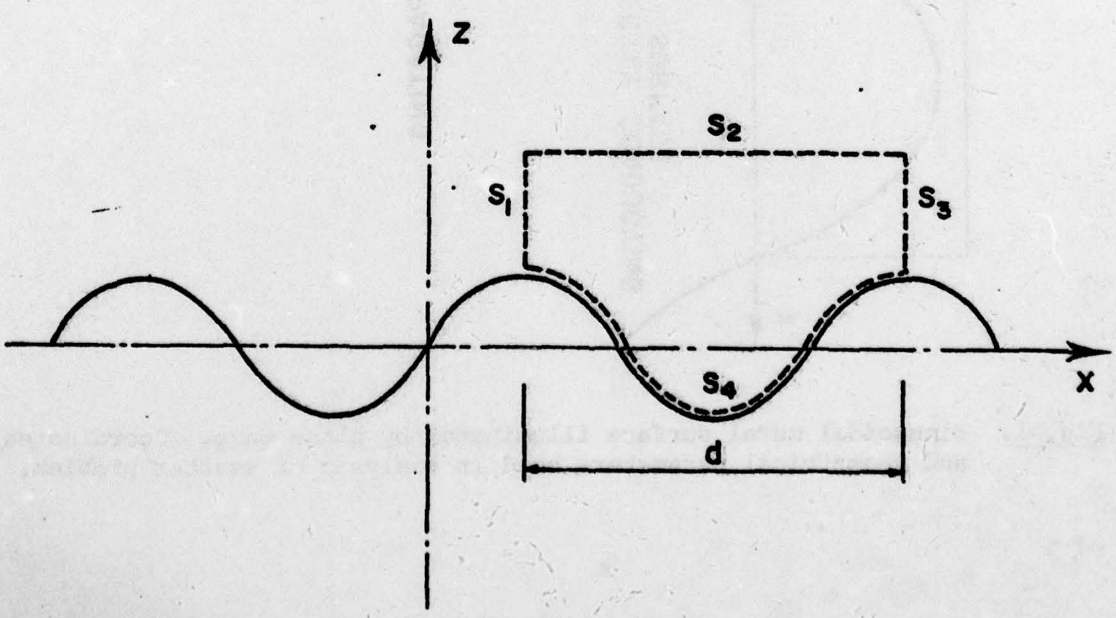


Fig. A2. Contour for formulations of reciprocity theorem.

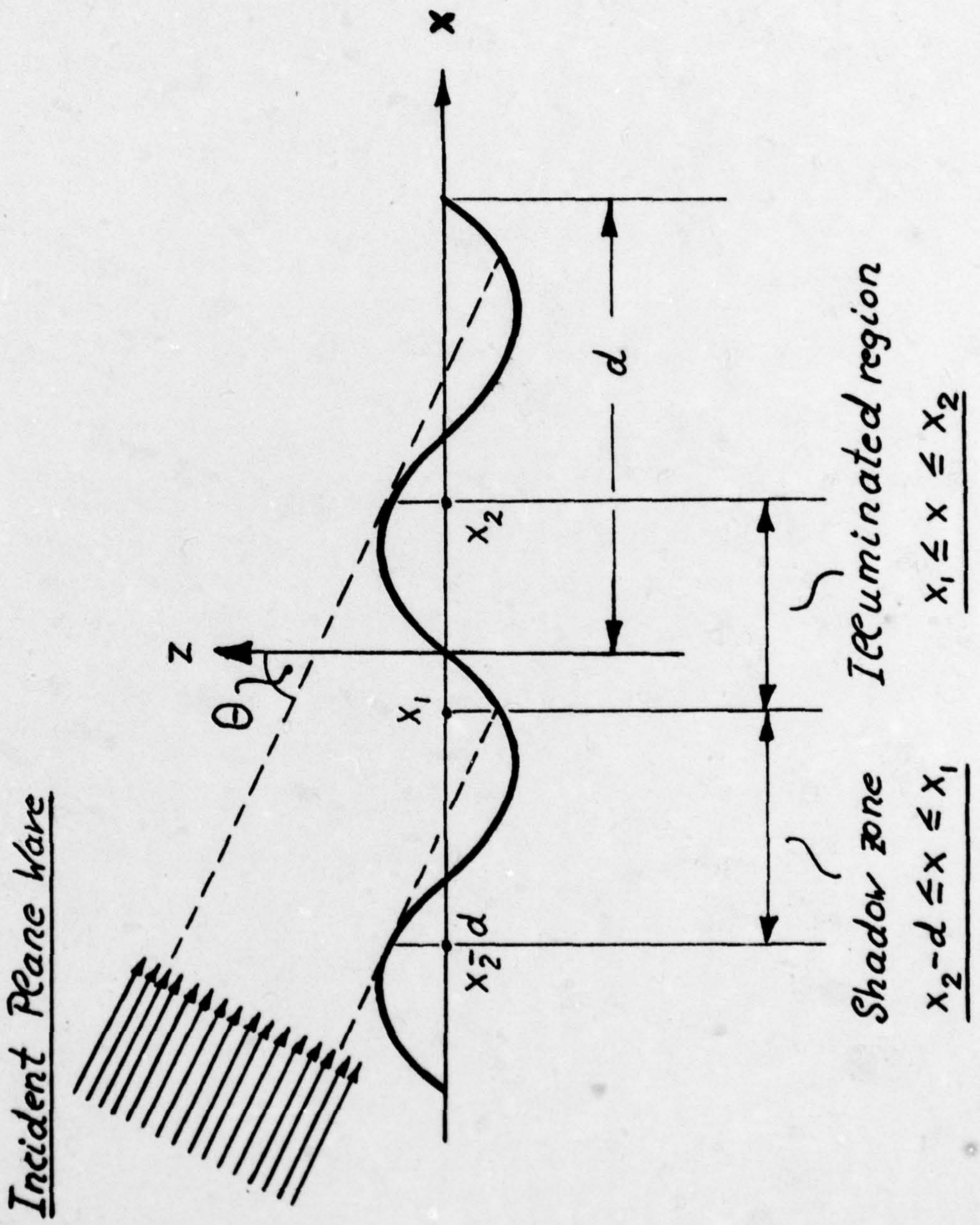


Fig B1. Illuminated and shadow zones of periodic metal surface (geometrical optics) at low arrival angles of incident plane wave

DEPARTMENT OF THE ARMY
HEADQUARTERS
UNITED STATES ARMY
Communications RESEARCH & DEVELOPMENT COMMAND
FORT MONMOUTH, NEW JERSEY 07703
ATTN DRDCO-COM-RM-4
OFFICIAL BUSINESS

U.S.MAIL

POSTAGE AND FEES PAID
DEPARTMENT OF THE ARMY
DOD-314